STABILITY OF A CONICAL COMBUSTION SURFACE WITH THE IGNITION OF A SOLID FUEL IN A SEMICLOSED VOLUME

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This article expounds a method for predicting stability, using the frequency characteristics of the fuel-feeding system or the structure of the central core, for units with a conical combustion surface. It is proposed to find these characteristics experimentally without ignition of the fuel.

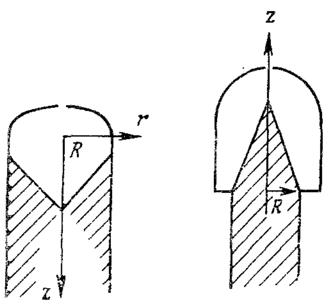
Schemes are known in which regulation of the gas coming from the solid fuel is achieved by a change in its surface. To this end, in the fuel there is placed a thin (in comparison with the radius R of the chamber) core, whose construction ensures the propagation of the flame along it at a velocity v_1 exceeding the normal velocity of the combustion, v. Under these circumstances, the surface of the fuel becomes conical, with an apex at the end of the central core (Fig. 1a). If a system of cores is used, R denotes the half-distance between adjacent cores. A conical surface can also be obtained with the ignition of a paste-form fuel extruded at a velocity v_1 from channels of radius R having lubricated walls ([4], see Fig. 1b).

In [5] there was evidently considered for the first time the possibility of autooscillations connected with distortion of the conical surface of a solid fuel, with its combustion in a semiclosed volume. The period t_0 of such oscillations is on the order of R/v. Since devices are used with a value of R from a few millimeters up to several centimeters, and $v \sim 1$ cm/sec, $t_0 \sim 10^{-1}$ sec. This, as a rule, is much greater than the characteristic combustion time t_1 of the heated layer (10^{-3}–10^{-2} sec). The radius of curvature of the surface with oscillations ($\sim$R) and the thickness of the heated layer, $\Delta x \sim 0.01$ cm, are in the same ratio. Consequently, it is possible to use the dependence of the normal combustion velocity on the pressure v(p), obtained under steady-state conditions with the combustion of a flat surface.

However, the characteristic time (triggering, lag) of the construction of the central core (based on the scheme of Fig. 1a) or of the fuel-feeding system (based on the scheme of Fig. 1b) can be comparable with the time t_0 of the reorganization of the surface, and v_1 will assume an "unsteady-state" value. Such a possibility was not considered in [5].

The motion of the burning surface, for schemes a and b of Fig. 1, respectively, is described by the equations

$$\partial z / \partial t = v\,[1 + (\partial z / \partial r)^2]^{1/2}$$
$$\partial z / \partial t = v_1 - v\,[1 + (\partial z / \partial r)^2]^{1/2} \qquad (1)$$



a b

Fig. 1

The system of coordinates is shown on Fig. 1. The boundary conditions for (1) are

$$r = 0,\quad \partial z / \partial t = v_1;\quad r = R,\quad \partial z / \partial t = 0 \qquad (2)$$

The steady-state solution has the form

$$z = \mathrm{const} + v_1 t - r\,[(v_1 / v)^2 - 1]^{1/2}$$
$$z = \mathrm{const} - r\,[(v_1 / v)^2 - 1]^{1/2} \qquad (3)$$

To determine the pressure and the form of the surface in the unsteady-state case, use must be made of the equality of the arrival G_+ and consumption G_- of the gas

$$G_+ = G_- \qquad (4)$$

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and the expression for the arrival

$$G_+ \sim \int_0^R (\partial z/\partial t)\, r dr, \qquad G_+ \sim \int_0^R (v_1 - \partial z/\partial t)\, r dr \tag{5}$$

The consumption is assumed to be a known function of the pressure. From (1)-(5), for the eigenfrequencies of small perturbations (of the type exp ωt), using linearization, we obtain

$$\begin{gathered} n^2\,[1 - (1+n)\exp(-n)]^{-1} = \beta \\ n^2\,[n - 1 + \exp(-n)]^{-1} = \beta \end{gathered} \tag{6}$$

Here

$$n = \omega t_0, \quad t_0 = [1 - (v/v_1)^2]^{-1/2} R/v, \quad \beta = 2\,\frac{d\ln v_1 - d\ln v}{d\ln G_- - c\ln v} \tag{7}$$

Stability of the form of the surface with respect to oscillations is ensured if the following inequality is satisfied for all the roots of (6):

$$\mathrm{Re}\, n < 0 \tag{8}$$

With $\beta = \beta(p)$, the numerical solution of (6), (8) gives the stability condition

$$-3.603 < \beta < 2, \ \beta < 2 \tag{9}$$

In the general case, in view of the above-mentioned "unsteady-state character" of v_1, the quantity β is a function of the complex dimensionless frequency n. Let the feeding system or the central core be subject to the action of oscillations of the pressure with an amplitude $|\Delta p|$ and of the variable frequency ε; for v_1, the amplitude $|\Delta v_1|$ and the phase shift φ are written as a function of ε:

$$\Delta v_1 = |\Delta v_1|\,(\Delta p / |\Delta p|)\, \exp(i\varphi) \tag{10}$$

In what follows, together with ε, it is convenient to use the quantity $m = \varepsilon t_0$, so that $\omega = i\varepsilon$, $n = im$. Then, from the definition of β (7), taking account of (10), it follows that

$$\begin{gathered} \nu = |\Delta\ln v_1 / \Delta\ln p|\,[\cos\varphi - (\mathrm{Re}\,\beta / \mathrm{Im}\,\beta)\sin\varphi] \\ \nu_- = \Delta\ln v_1 / \Delta\ln p\,|\,[\cos\varphi - (\sin\varphi)(\mathrm{Re}\,\beta - 1)/\mathrm{Im}\,\beta] \\ (\nu = d\ln v / d\ln p, \quad \nu_- = d\ln G_- / d\ln p) \end{gathered} \tag{11}$$

Setting n = im at the boundary of the oscillational instability, and equating the individual real and imaginary parts, from (6) we obtain

$$\begin{gathered} \mathrm{Re}\,\beta = m^2 a\,(a^2 + b^2)^{-1}, \ \mathrm{Im}\,\beta = m^2 b\,(a^2 + b^2)^{-1} \\ a = m\sin m + \cos m - 1, \ b = \sin m - m\cos m \\ \mathrm{Re}\,\beta = m^2 c\,(c^2 + d^2)^{-1}, \ \mathrm{Im}\beta = m^2 d\,(c^2 + d^2)^{-1} \\ c = 1 - \cos m, \qquad d = m - \sin m \end{gathered} \tag{12}$$

Thus, the right-hand parts of (11) are known functions of m, which permits constructing the limit of stability in the form of a parametric (with the parameter m) dependence $\nu(\nu_-)$. In practice, it is always true that $\nu_- = 1$, but the construction of $\nu(\nu_-)$ is a more reliable way to obtain the values of $\nu(1)$ at the limit of stability.

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